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Magnon like solutions for strings in I-brane background

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Abstract: We study the solutions for fundamental string rotating in a background generated by a 1+1 dimensional intersection of two orthogonal stacks of fivebranes in type IIB string theory. We show the existence of magnon like solutions for the string moving simultaneously in the two spheres in this background and find the relevant dispersion relation among the various conserved charges.

Keywords: D-branes, AdS-CFT Correspondence.

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Contents

1. Introduction and summary

AdS/CFT correspondence [1] relates the spectrum of free strings on $\mathrm{AdS}_5\times \mathrm{S}^5$ with that of the spectrum of operator dimensions in the $\mathcal{N} = 4$ super Yang-Mills (SYM) in four dimensions. This mapping is highly nontrivial and challenging. A better understanding of this mapping will be to look at both the gauge and gravity theories at certain limits such as large angular momentum limit and then compare the spectrum. In the understanding the gauge/gravity duality, an interesting observation is that the $\mathcal{N} = 4$ SYM theory can be described by the integrable spin chain model where the anomalous dimension of the gauge invariant operators were found [2−8]. It was further noticed that the string theory also has as integrable structure in the semiclassical limit and the anomalous dimension in the $\mathcal{N} = 4$ SYM can be derived from the relation between conserved charges of the worldsheet solitonic string solution on the dual string theory on $\mathrm{AdS}_5\times \mathrm{S}^5$. In this connection, Hofman and Maldacena (HM) [9] considered a special limit where the problem of determining the spectrum of both sides becomes rather simple. The spectrum consists of an elementary excitation known as magnon which propagate with a conserved momentum p along the long spin chain. In the dual formulation, the most important ingredient is semiclassical string solutions, which can be mapped to long trace operator with large energy and large angular momenta. Once this connection was established, there was a lot of work devoted to understanding of this correspondence (see for example [10]–[26]).

To understand the AdS/CFT like correspondence in a more general background, it is interesting to find out dispersion relation among various conserved charges in case of classical rotating strings in the gravity side and then to look for the corresponding operators in the dual theory. For example, the magnon like dispersion relation in NS5-brane background was considered in [27].

An interesting background configuration is the I-brane background [28], which is the intersection of two stacks of NS5-branes in type II string theory on $R^{1,1}$. When all the five branes are coincident, the near horizon geometry of this configuration is given by [28][1]

$$R^{2,1} \times R_\phi \times \mathrm{SU}(2)_{k_1} \times \mathrm{SU}(2)_{k_2}, \tag{1.1}$$

[1] See [29] for related background and for the gauge theory results.

where R_ϕ is one combination of two radial directions away from two sets of NS5-branes, k_1 and k_2 are the number of NS5-branes in each stack. The coordinates of $R^{2,1}$ are x^0, x^1 and one more combination of two radial directions. The two SU(2)s corresponding to two angular three spheres corresponding to $(R^4)_{2345}$ and $(R^4)_{6789}$. The very fact that (1.1) is an exact solution to string equations of motion allows us to obtain information about the intersecting brane system more, which is not accessible via gauge theory. As it is clear from (1.1) the exhibits a higher symmetry than the full intersecting brane configuration. In particular, the combination of radial directions away from the intersection that enters $R^{2,1}$ appears symmetrically with the other spatial directions, and the background has a higher Poincare symmetry, ISO(2, 1), than the expected ISO(1, 1). The holographic mapping between field theory living on the I-brane and the corresponding bulk theory was studied in [28].[2] On the other hand it is also well known from the study of AdS/CFT correspondence that it is possible to derive more information about the boundary CFT theory from the study of the semiclassical string and D1-brane configurations in the bulk of AdS_5.[3]

Motivated by the recent developments in the AdS/CFT correspondence, we will investigate the classical string dynamics in the I-brane background to understand the rotating string solutions. These string solutions correspond to multispin string solutions on the I-brane background and solitonic string solution in the worldsheet point of view. Due to the lack of sufficient knowledge about the theory on the worldvolume of the I-brane we can't compare them with the dual theory. However the knowledge about the semiclassical rotating string on the I-brane certainly gives information about the possible nature of operators in some dual theory.

In this background, the solitonic string solution which are static and uniformly wrapping the two transverse spheres, and the dispersion relation among various conserved charges were studied in [43]. Here, we will generalize this solution to the rotating one in the transverse spheres and find the dispersion relation among various charges. To do so, we will first solve the equations of motion for a string rotating simultaneously on both the spheres and then by using the Virasoro constraints the dispersion relation will be expressed in terms of various conserved charges that the background obeys in general. Usually, in case of the giant magnon on $R \times S^3$, the energy and one of the angular momenta for one giant magnon are infinite but their difference is finite. In the I-brane background, on the other hand, which has also $R \times S^3$ or $R \times S^3 \times S^3$, the string soliton is composed of multiple magnon-like solutions and one of them, we call this magnon-like solution, has a similar shape to the giant magnon on $R \times S^3$. However, the dispersion relation for magnon-like one is different in that it contains additional linear momenta in two radial directions and is described by the finite conserved charges. We would like to note that on the I-brane background all conserved charges are regularized, so have finite values. If we choose the range of the world sheet string coordinate as $-\infty < \sigma < \infty$ like HM case [9] or $-\pi/2 < \sigma < \pi/2$, the string soliton becomes a combination of infinite magnon-like solutions or finite numbers. For the closed string case, since $-\pi < \sigma < \pi$ the solitonic string is given by the finite

[2]For some relevant works, see [30 – 39]

[3]For review, see [40 – 42].

numbers of the magnon-like solutions.

We would like to mention that in general getting magnon/spike solutions in I-brane background is rather cumbersome. However, we will present in this paper, a parameter space of solutions where there exists a magnon like shape when we restrict the motion of the string along both the spheres. It would be certainly interesting to find out more general rotating string solutions in this backgrounds. A greater challenge will be to find out the dual operators in the worldvolume theory which correspond to the semiclassical string solutions presented in this paper.

The rest of the paper is organized as follows. In section 2, we present the background solution corresponding to the intersection of two five-branes on $R^{1,1}$ in type IIB string theory and study the equations of motion and Virasoro constraints of the fundamental string rotating simultaneously along both the spheres. Section-3 devoted to the study the rotating string solution interpreted as a single magnon like solution while the motion is restricted to only one sphere. We present the corresponding dispersion relation along various charges and interpret that solution as a single magnon solution. Further we present a more general solution when the string moves simultaneously along both the spheres. We analyze the results in a particular parameter space of solutions. Finally in section-4, we present our conclusions.

2. F-string in the background of I-brane

In this section we will study the dynamics of fundamental string in the background studied in [28]. Namely, we consider the intersection of two stack of NS5-branes on $R^{1,1}$. We have k_1 number of NS5-branes extended in $(0,1,2,3,4,5)$ directions and another set of k_2 number of NS5-branes extended in $(0,1,6,7,8,9)$ directions. Let us define

$$\begin{aligned} \mathbf{y} &= (x^2, x^3, x^4, x^5)\,, \\ \mathbf{z} &= (x^6, x^7, x^8, x^9)\ . \end{aligned} \tag{2.1}$$

We have k_1 NS5-branes localized at the points $\mathbf{z}_n$ $n = 1, \dots, k_1$, and k_2 NS5-branes localized at the points $\mathbf{y}_a\,, a = 1\dots, k_2$. Every pairs of fivebranes from different sets intersect at different point $(\mathbf{y}_a, \mathbf{z}_n)$. The supergravity background corresponding to this configuration takes the form

$$\begin{aligned} \Phi(\mathbf{z},\mathbf{y}) &= \Phi_1(\mathbf{z}) + \Phi_2(\mathbf{y})\,, \\ g_{\mu\nu} &= \eta_{\mu\nu}\,,\ \ (\mu,\nu = 0,1)\,, \\ g_{\alpha\beta} &= e^{2(\Phi_2 - \Phi_2(\infty))}\delta_{\alpha\beta}, \\ \mathcal{H}_{\alpha\beta\gamma} &= -\epsilon_{\alpha\beta\gamma\delta}\partial^\delta\Phi_2\,,\ \ (\alpha,\beta,\gamma,\delta = 2,3,4,5)\,, \\ g_{pq} &= e^{2(\Phi_1 - \Phi_1(\infty))}\delta_{pq}\,, \\ \mathcal{H}_{pqr} &= -\epsilon_{pqrs}\partial^s\Phi_1\,,\ \ \ (p,q,r,s = 6,7,8,9)\,, \end{aligned} \tag{2.2}$$

where Φ on the first line means the dilaton and where

$$e^{2(\Phi_1-\Phi_1(\infty))} = 1 + \sum_{n=1}^{k_1} \frac{l_s^2}{|\mathbf{z}-\mathbf{z}_n|^2} \, ,$$
$$e^{2(\Phi_2-\Phi_2(\infty))} = 1 + \sum_{a=1}^{k_2} \frac{l_s^2}{|\mathbf{y}-\mathbf{y}_a|^2} \, . \tag{2.3}$$

Our goal is to find solutions for rotating string in this background when $\mathbf{z}_n = \mathbf{y}_a = 0$. To simplify our notation let us denote

$$e^{2(\Phi_1-\Phi_1(\infty))} = H_1(\mathbf{z}) \, , \quad e^{2(\Phi_2-\Phi_2(\infty))} = H_2(\mathbf{y}) \, , \tag{2.4}$$

where for coincident branes we have

$$H_1 = 1 + \frac{k_1 l_s^2}{|\mathbf{z}|^2} \, , \quad H_2 = 1 + \frac{k_2 l_s^2}{|\mathbf{y}|^2} \, . \tag{2.5}$$

Let us now consider the probe brane in the near horizon limit where

$$\frac{k_1 l_s^2}{|\mathbf{z}|^2} \gg 1 \, , \quad \frac{k_2 l_s^2}{|\mathbf{y}|^2} \gg 1 \tag{2.6}$$

so that we can write

$$H_1 = \frac{\lambda_1}{r_1^2} \, , \quad \lambda_1 = k_1 l_s^2 \, , \quad H_2 = \frac{\lambda_2}{r_2^2} \, , \quad \lambda_2 = k_2 l_s^2 \, . \tag{2.7}$$

Then the metric takes the form

$$ds^2 = -dt^2 + \frac{\lambda_1}{r_1^2} dr_1^2 + \frac{\lambda_2}{r_2^2} dr_2^2 + \lambda_1 d\Omega_1^{(3)} + \lambda_2 d\Omega_2^{(3)} \, , \tag{2.8}$$

where $d\Omega_1^{(3)}$ and $d\Omega_2^{(3)}$ correspond to the line elements on the unit sphere. To describe them better we introduce the following coordinates

$$x^2 + ix^3 = r_1 \cos\theta_1 e^{i\phi_1} \, , \qquad x^4 + ix^5 = r_1 \cos\theta_1 e^{i\psi_1} \, ,$$
$$x^6 + ix^7 = r_2 \cos\theta_2 e^{i\phi_2} \, , \qquad x^8 + ix^9 = r_2 \cos\theta_2 e^{i\psi_2} \tag{2.9}$$

so that

$$\begin{aligned}
d\Omega_1^{(3)} &= d\theta_1^2 + \sin^2\theta_1 d\phi_1^2 + \cos^2\theta_1 d\psi_1^2, \\
b_{\phi_1\psi_1} &= \lambda_1 \cos^2\theta_1 \, , && 0 < \theta_1 < \frac{\pi}{2} \, , \quad 0 = \phi_1, \psi_1 < 2\pi \, , \\
d\Omega_2^{(3)} &= d\theta_2^2 + \sin^2\theta_2 d\phi_2^2 + \cos^2\theta_2 d\psi_2^2, \\
b_{\phi_2\psi_2} &= \lambda_2 \cos^2\theta_2 \, , && 0 < \theta_2 < \frac{\pi}{2} , \quad 0 = \phi_2, \psi_2 < 2\pi \, .
\end{aligned} \tag{2.10}$$

As usual our starting point is the Polyakov form of the string action in the background (2.8)

$$S = -\frac{1}{4\pi\alpha'}\int_{-\pi/2}^{\pi/2} d\sigma d\tau [\sqrt{-\gamma}\gamma^{\alpha\beta} g_{MN}\partial_\alpha x^M \partial_\beta x^N - e^{\alpha\beta}\partial_\alpha x^M \partial_\beta x^N b_{MN}] + \\ + \frac{1}{4\pi}\int_{-\pi/2}^{\pi/2} d\sigma d\tau \sqrt{-\gamma} R\Phi \,, \quad (2.11)$$

where $\gamma^{\alpha\beta}$ is a world-sheet metric and R is its Ricci scalar. Further, $e^{\alpha\beta}$ is defined as $e^{01} = -e^{10} = 1$. Finally, the modes $x^M, M = 0, \dots, 9$ parameterize the embedding of the string in the background (2.8). The variation of the action (2.11) with respect to x^M implies following equations of motion

$$-\frac{1}{4\pi\alpha'}\sqrt{-\gamma}\gamma^{\alpha\beta}\partial_K g_{MN}\partial_\alpha x^M \partial_\beta x^N + \frac{1}{2\pi\alpha'}\partial_\alpha[\sqrt{-\gamma}\gamma^{\alpha\beta} g_{KM}\partial_\beta x^M] - \\ -\frac{1}{2\pi\alpha'}\partial_\alpha[\epsilon^{\alpha\beta}\partial_\beta x^M b_{KM}] + \frac{1}{4\pi\alpha'}\epsilon^{\alpha\beta}\partial_\alpha x^M \partial_\beta x^N \partial_K b_{MN} + \frac{1}{4\pi}\partial_K \Phi\sqrt{-\gamma} R = 0 \,. \quad (2.12)$$

Finally the variation of the action with respect to the metric implies the constraints

$$-\frac{4\pi}{\sqrt{-\gamma}}\frac{\delta S}{\delta\gamma^{\alpha\beta}} = \frac{1}{\alpha'} g_{MN}\partial_\alpha x^M \partial_\beta x^N - R_{\alpha\beta} + \\ + (\nabla_\alpha\nabla_\beta x^M)\partial_M \Phi + (\partial_\alpha x^M \partial_\beta x^N)\partial_M\partial_N\Phi - \\ -\frac{1}{2}\gamma_{\alpha\beta}\left(\frac{1}{\alpha'}\gamma^{\gamma\delta}\partial_\gamma x^M \partial_\delta x^N g_{MN} - R\Phi + 2\nabla^\alpha\nabla_\alpha\Phi\right) . \quad (2.13)$$

As the first step let us introduce two modes ρ_1 and ρ_2 defined through the relations

$$r_1 = e^{\frac{\rho_1}{\sqrt{\lambda_1}}} \,, \quad r_2 = e^{\frac{\rho_2}{\sqrt{\lambda_2}}} \,. \quad (2.14)$$

Then, following [28] we introduce two modes r, y through the relation

$$Qr = \frac{1}{\sqrt{\lambda_1}}\rho_1 + \frac{1}{\sqrt{\lambda_2}}\rho_2 \,, \quad Qy = \frac{1}{\sqrt{\lambda_2}}\rho_1 - \frac{1}{\sqrt{\lambda_1}}\rho_2 \,, \quad (2.15)$$

where

$$Q = \frac{1}{\sqrt{\lambda}} \,, \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \,. \quad (2.16)$$

Note that the inverse transformations of (2.15) take the forms

$$\rho_1 = \frac{1}{\sqrt{\lambda_1+\lambda_2}}\left(\sqrt{\lambda_1} y + \sqrt{\lambda_2} r\right) \,, \\ \rho_2 = \frac{1}{\sqrt{\lambda_1+\lambda_2}}\left(\sqrt{\lambda_1} r - \sqrt{\lambda_2} y\right) \,. \quad (2.17)$$

Note that this result implies that the dilaton is a function of r only

$$\Phi = \Phi_1 + \Phi_2 = \frac{1}{2}(H_1 + H_2) + \Phi_1(\infty) + \Phi_2(\infty) = \\ = -\frac{1}{\sqrt{\lambda_1}}\rho_1 - \frac{1}{\sqrt{\lambda_2}}\rho_2 + \Phi_0 = -Qr + \Phi_0 \,. \quad (2.18)$$

With the help of the variables r, y the action for string in I-brane background takes the form

$$S = -\frac{1}{4\pi\alpha'}\int_{-\pi/2}^{\pi/2} d\sigma d\tau [\sqrt{-\gamma}\gamma^{\alpha\beta}(-\partial_\alpha t\partial_\beta t + \partial_\alpha r\partial_\beta r + \partial_\alpha y\partial_\beta y + \\ + g_{mn}\partial_\alpha x^m\partial_\beta x^n - e^{\alpha\beta}\partial_\alpha x^m\partial_\beta x^n b_{mn}] - \frac{1}{4\pi}\int_{-\pi/2}^{\pi/2} d\sigma d\tau\sqrt{-\gamma}RQr\,, \quad (2.19)$$

where $x^{m,n}$ label angular coordinates corresponding to S^3_1, S^3_2 respectively.

Looking at the form of the background (2.8) and (2.10) we observe that the action (2.19) is invariant under following transformations of fields

$$\begin{aligned} t'(\tau,\sigma) &= t(\sigma,\tau) + \epsilon_t\,, \\ y'(\tau,\sigma) &= y(\tau,\sigma) + \epsilon_y\,, \\ \psi'_1(\tau,\sigma) &= \psi_1(\tau,\sigma) + \epsilon_{\psi_1}\,, \\ \psi'_2(\tau,\sigma) &= \psi_2(\tau,\sigma) + \epsilon_{\psi_2}\,, \\ \phi'_1(\tau,\sigma) &= \phi_1(\tau,\sigma) + \epsilon_{\phi_1}\,, \\ \phi'_2(\tau,\sigma) &= \phi_2(\tau,\sigma) + \epsilon_{\phi_2}\,, \end{aligned} \quad (2.20)$$

where $\epsilon_t, \epsilon_y, \epsilon_{\phi_1}, \epsilon_{\phi_2}, \epsilon_{\psi_1}, \epsilon_{\psi_2}$ are constants. Then it is straightforward to determine corresponding conserved charges

$$\begin{aligned} P_t &= -\frac{1}{2\pi\alpha'}\int_{-\pi/2}^{\pi/2} d\sigma\sqrt{-\gamma}\gamma^{\tau\alpha}\partial_\alpha t\,, \\ P_{\psi_1} &= \frac{1}{2\pi\alpha'}\int_{-\pi/2}^{\pi/2} d\sigma[\sqrt{-\gamma}\gamma^{\tau\alpha}g_{\psi_1\psi_1}\partial_\alpha\psi_1 - \partial_\sigma\phi_1 b_{\phi_1\psi_1}]\,, \\ P_{\psi_2} &= \frac{1}{2\pi\alpha'}\int_{-\pi/2}^{\pi/2} d\sigma[\sqrt{-\gamma}\gamma^{\tau\alpha}g_{\psi_2\psi_2}\partial_\alpha\psi_2 - \partial_\sigma\phi_2 b_{\phi_2\psi_2}]\,, \\ P_{\phi_1} &= \frac{1}{2\pi\alpha'}\int_{-\pi/2}^{\pi/2} d\sigma[\sqrt{-\gamma}\gamma^{\tau\alpha}g_{\psi_1\psi_1}\partial_\alpha\psi_1 + \partial_\sigma\psi_1 b_{\phi_1\psi_1}]\,, \\ P_{\phi_2} &= \frac{1}{2\pi\alpha'}\int_{-\pi/2}^{\pi/2} d\sigma[\sqrt{-\gamma}\gamma^{\tau\alpha}g_{\psi_2\psi_2}\partial_\alpha\psi_2 + \partial_\sigma\psi_2 b_{\phi_2\psi_2}]\,, \\ P_y &= \frac{1}{2\pi\alpha'}\int_{-\pi/2}^{\pi/2} d\sigma[\sqrt{-\gamma}\gamma^{\tau\alpha}g_{yy}\partial_\alpha y]\,. \end{aligned} \quad (2.21)$$

Note that P_t is related to the energy as $P_t = -E$. In [43] the homogeneous string and D1-brane solutions in I-brane background have been studied.[4] It was argued that it is necessary to find the configuration when string moves on both two spheres simultaneously. For that reason we have to consider an ansatz where string moves simultaneously on both

[4]Dynamics of D1-brane probe in given background was studied in [39, 38].

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the spheres S_1^3 and S_2^3 as follows

$$\begin{aligned} t &= t(\tau), & r &= r(\tau), & y &= y(\tau), \\ \theta_1 &= \theta_1(m)\,, & \psi_1 &= \omega_1\tau + g_1(m)\,, & \phi_1 &= \nu_1\tau + h_1(m)\,, \\ \theta_2 &= \theta_2(m)\,, & \psi_2 &= \omega_2\tau + g_2(m)\,, & \phi_2 &= \nu_2\tau + h_2(m)\,, \end{aligned} \tag{2.22}$$

where

$$m = \alpha\sigma + \beta\tau \tag{2.23}$$

and we also consider solution in the conformal gauge $\gamma^{\alpha\beta} = \eta^{\alpha\beta}$. In this gauge the constraints (2.13) that now follow from the variation of the action (2.19) take simpler forms

$$\begin{aligned} T_{\sigma\sigma} &= -4\pi\frac{\delta S}{\delta\gamma^{\sigma\sigma}} = \frac{1}{2\alpha'}(g_{MN}\partial_\sigma x^M\partial_\sigma x^N + g_{MN}\partial_\tau x^M\partial_\tau x^N) - Q\partial_\tau^2\rho\,, \\ T_{\tau\tau} &= -4\pi\frac{\delta S}{\delta\gamma^{\tau\tau}} = \frac{1}{2\alpha'}(g_{MN}\partial_\sigma x^M\partial_\sigma x^N + g_{MN}\partial_\tau x^M\partial_\tau x^N) - Q\partial_\sigma^2\rho\,, \\ T_{\tau\sigma} &= -4\pi\frac{\delta S}{\delta\gamma^{\sigma\tau}} = \frac{1}{\alpha'}g_{MN}\partial_\sigma x^M\partial_\tau x^N - Q\partial_\sigma\partial_\tau\rho\,. \end{aligned} \tag{2.24}$$

Further, in conformal gauge the equations of motion for t, y, r take the form

$$\partial_\alpha[\eta^{\alpha\beta}\partial_\beta t] = 0\,, \quad \partial_\alpha[\eta^{\alpha\beta}\partial_\beta r] = 0\,, \quad \partial_\alpha[\eta^{\alpha\beta}\partial_\beta y] = 0 \tag{2.25}$$

that have solutions

$$t = \kappa\tau\,, \quad r = v_r\tau + r_0\,, \quad y = v_y\tau + y_0 \tag{2.26}$$

for constants $\kappa, v_y, v_r, r_0, y_0$.

Now we are going to study the dynamics of fundamental strings on both three S_1^3, S_2^3. We start then with the equations of motion for ϕ_1 and ψ_1. It can be easily shown that these equations imply two differential equations

$$\begin{aligned} h_1' &= \frac{1}{\lambda_1\sin^2\theta_1(\alpha^2-\beta^2)}(C_1 - \lambda_1\alpha\omega_1\cos^2\theta_1 + \lambda_1\beta\nu_1\sin^2\theta_1)\,, \\ g_1' &= \frac{1}{(\alpha^2-\beta^2)\lambda_1\cos^2\theta_1}(D_1 + \lambda_1\beta\omega_1\cos^2\theta_1 + \lambda_1\nu_1\alpha\cos^2\theta_1)\,. \end{aligned} \tag{2.27}$$

where $h'(m) = \frac{dh}{dm}$ and where C_1, D_1 are integration constants. In the same way we consider the equation of motion for ψ_2, ϕ_2 with the result

$$\begin{aligned} h_2' &= \frac{1}{(\alpha^2-\beta^2)\lambda_2\sin^2\theta_1}(C_2 + \lambda_2\sin^2\theta_2\beta\nu_2 - \omega_2\alpha\lambda_2\cos^2\theta_2)\,, \\ g_2' &= \frac{1}{(\alpha^2-\beta^2)\lambda_2\cos^2\theta_2}(D_2 + \lambda_2\beta\omega_2\cos^2\theta_2 + \lambda_2\nu_2\alpha\cos^2\theta_2)\,, \end{aligned} \tag{2.28}$$

where again C_2, D_2 are integration constants. Let us now start to solve the Virasoro constraints for the given model. The constraint $T_{\tau\sigma} = 0$ implies

$$\begin{aligned} &g_{\phi_1\phi_1}\alpha h_1'(\nu_1 + \beta h_1') + g_{\psi_1\psi_1}\alpha g_1'(\omega_1 + \beta g_1') + g_{\theta_1\theta_1}\alpha\beta\theta_1'^2 + \\ &+ g_{\phi_2\phi_2}\alpha h_2'(\nu_2 + \beta h_2') + g_{\psi_2\psi_2}\alpha g_2'(\omega_2 + \beta g_2') + g_{\theta_2\theta_2}\alpha\beta\theta_2'^2 = 0\,. \end{aligned} \tag{2.29}$$

On the other hand the Virasoro constraints $T_{\tau\tau} = T_{\sigma\sigma} = 0$ take the form

$$0 = g_{\phi_1\phi_1}[(\nu_1 + \beta h_1')^2 + \alpha^2 h_1'^2] + g_{\psi_1\psi_1}[(\omega_1 + \beta g_1')^2 + \alpha^2 g_1'^2] + g_{\theta_1\theta_1}(\alpha^2 + \beta^2)\theta_1'^2$$
$$+ g_{\phi_2\phi_2}[(\nu_2 + \beta h_2')^2 + \alpha^2 h_2'^2] + g_{\psi_2\psi_2}[(\omega_2 + \beta g_2')^2 + \alpha^2 g_2'^2] + g_{\phi_2\phi_2}(\alpha^2 + \beta^2)\theta_2'^2$$
$$-\kappa^2 + v_r^2 + v_y^2 \ . \tag{2.30}$$

Now, if we combine these constraints as $-\frac{(\alpha^2+\beta^2)}{\alpha\beta} T_{\sigma\tau} + T_{\tau\tau}$ we obtain

$$0 = -\nu_1 C_1 - \omega_1 D_1 - \nu_2 C_2 - \omega_2 D_2 + \beta(-\kappa^2 + v_r^2 + v_y^2) \ . \tag{2.31}$$

Then if we use (2.27), (2.28) together with (2.31) in the constraint $T_{\tau\tau} = 0$ we obtain

$$(\beta^2 + \alpha^2)(\lambda_1 \theta_1'^2 + \lambda_2 \theta_2'^2)$$
$$= \frac{(\alpha^2+\beta^2)^2}{(\alpha^2-\beta^2)^2}(\kappa^2 - v_r^2 - v_y^2) - \frac{(\alpha^2+\beta^2)\alpha^2}{(\alpha^2-\beta^2)^2}(g_{\phi_1\phi_1}\nu_1^2 + g_{\psi_1\psi_1}\omega_1^2)\left(\frac{g_{\phi_1\phi_1}g_{\psi_1\psi_1} + b_{\phi_1\psi_1}^2}{g_{\phi_1\phi_1}g_{\psi_1\psi_1}}\right)$$
$$-\frac{\alpha^2+\beta^2}{(\alpha^2-\beta^2)^2}\left[\frac{C_1^2}{g_{\phi_1\phi_1}} + \frac{D_1^2}{g_{\psi_1\psi_1}} + +2\alpha\frac{b_{\phi_1\psi_1}}{g_{\psi_1\psi_1}g_{\phi_1\phi_1}}(D_1\nu_1 g_{\phi_1\phi_1} - C_1\omega_1 g_{\psi_1\psi_1})\right]$$
$$-\frac{(\alpha^2+\beta^2)\alpha^2}{(\alpha^2-\beta^2)^2}(g_{\phi_2\phi_2}\nu_2^2 + g_{\psi_2\psi_2}\omega_2^2)\left(\frac{g_{\phi_2\phi_2}g_{\psi_2\psi_2} + b_{\phi_2\psi_2}^2}{g_{\phi_2\phi_2}g_{\psi_2\psi_2}}\right) -$$
$$-\frac{\alpha^2+\beta^2}{(\alpha^2-\beta^2)^2}\left[\frac{C_2^2}{g_{\phi_2\phi_2}} + \frac{D_2^2}{g_{\psi_2\psi_2}} + 2\alpha\frac{b_{\phi_2\psi_2}}{g_{\psi_2\psi_2}g_{\phi_2\phi_2}}(D_2\nu_2 g_{\phi_2\phi_2} - C_2\omega_2 g_{\psi_2\psi_2})\right] \tag{2.32}$$

This differential equation determines the most general evolutions of θ's. As it is clear from the above, solving for general θ is quite hard. Hence, in what follows we will analyze it in some special situations.

2.1 Magnon solutions in $R \times S_1^3$

Let us start with the situation when $\theta_2' = h_2' = g_2' = 0$, $\omega_2 = \nu_2 = 0$. For our convenience, we set $D_1 = \lambda_1 d$, $C_1 = \lambda_1 c$ and $\kappa^2 - v_r^2 - v_y^2 = \lambda_1\gamma$. Then, the above equation takes the form

$$\theta_1'^2 = \frac{1}{(\alpha^2-\beta^2)^2}\left[(\alpha^2+\beta^2)\gamma - \alpha^2(\nu_1^2 - \omega_1^2) - 2\alpha(d\nu_1 + c\omega_1) - \frac{(\alpha\omega_1 - c)^2}{\sin^2\theta_1} - \frac{d^2}{\cos^2\theta_1}\right] . \tag{2.33}$$

Let us write the above differential equation in the following form

$$\theta_1'^2 = A^2 - \frac{B^2}{\sin^2\theta_1} - \frac{d'^2}{\cos^2\theta_1} \ , \tag{2.34}$$

where

$$A^2 = \frac{1}{(\alpha^2-\beta^2)^2}\left[(\alpha^2+\beta^2)\gamma - \alpha^2(\nu_1^2 - \omega_1^2) - 2\alpha(d\nu_1 + c\omega_1)\right] ,$$
$$B^2 = \frac{1}{(\alpha^2-\beta^2)^2}\left[(\alpha\omega_1 - c)^2\right] , d'^2 = \frac{d^2}{(\alpha^2-\beta^2)^2} \ . \tag{2.35}$$

Then

$$\theta_1' = A\,\frac{\sqrt{(\sin^2\theta_1 - \sin^2\theta_{\rm min})(\sin^2\theta_{\rm max} - \sin^2\theta_1)}}{\sin\theta_1\cos\theta_1}\,, \tag{2.36}$$

where

$$\begin{aligned}\sin^2\theta_{\rm max} &= \frac{(A^2+B^2-d'^2)+\sqrt{(A^2+B^2-d'^2)^2-4A^2B^2}}{2A^2}\,,\\ \sin^2\theta_{\rm min} &= \frac{(A^2+B^2-d'^2)-\sqrt{(A^2+B^2-d'^2)^2-4A^2B^2}}{2A^2}\,.\end{aligned} \tag{2.37}$$

Let us try to find the solution when $\sin^2\theta_{\rm max} = 1$. This occurs when

$$D_1 = 0\,, \quad \text{with } \sin\theta_{\rm min} = \frac{B}{A}\,. \tag{2.38}$$

The final form of θ_1' is

$$\theta_1' = A\frac{\sqrt{\sin^2\theta_1 - \sin^2\theta_{\rm min}}}{\sin\theta_1}\,. \tag{2.39}$$

Note that the range of $\theta_{1,min} \le \theta_1 \le \frac{\pi}{2}$ corresponds to the half of a magnon-like solution. For the convenience, we call one element of the solitonic string solution having the magnon shape as a magnon-like solution. Actually, the solution obtained here is a combination of these magnon-like solution. So the number of the magnon-like solution contained in the solitonic string solution is given by the followings. If the range of the string world sheet is given by $-\frac{\pi}{2} \le \sigma \le \frac{\pi}{2}$ for an open string, the number of magnon-like solution is determined by

$$\pi = \int_{-\pi/2}^{\pi/2} d\sigma = \frac{2n}{\alpha}\int_{\theta_{1,min}}^{\pi/2}\frac{d\theta_1}{\theta_1'}, \tag{2.40}$$

where n means the number of the magnon-like solutions. After calculating the last equation, the number of the magnon-like solutions is given by $n = \alpha A$.

From now on, we will concentrate on the conserved quantities for magnon-like solutions which will give a dispersion relation for one magnon-like solution. Using the definitions of the conserved charges (2.21) they can be rewritten as the integral form over θ_1

$$P_t = \frac{1}{\pi\alpha'}\frac{\kappa}{\alpha}\,I, \tag{2.41}$$

$$P_y = \frac{1}{\pi\alpha'}\frac{v_y}{\alpha}\,I, \tag{2.42}$$

$$P_r = \frac{1}{\pi\alpha'}\frac{v_r}{\alpha}\,I, \tag{2.43}$$

$$P_{\phi_1} = -\frac{\lambda_1}{\pi\alpha'}\frac{1}{\alpha^2-\beta^2}\left(\frac{\beta}{\alpha}c + \alpha\nu_1\right)\,I, \tag{2.44}$$

$$P_{\psi_1} = -\frac{\lambda_1}{\pi\alpha'}\frac{c-\alpha\omega_1}{\alpha^2-\beta^2}\left(I - I'\right), \tag{2.45}$$

where

$$\begin{aligned} I &= \int_{\theta_{\rm min}}^{\pi/2} d\theta_1\,\frac{1}{\theta_1'} = \frac{\pi}{2A}\,,\\ I' &= \int_{\theta_{\rm min}}^{\pi/2} d\theta_1\,\frac{1}{\theta_1'\sin^2\theta_1} = \frac{\pi}{2B}\,.\end{aligned} \tag{2.46}$$

The angle difference in the ϕ_1-direction is given by

$$\Delta\phi_1 = \frac{2}{\alpha^2-\beta^2}\left[(c-\alpha\omega_1)I' + (\alpha\omega_1+\beta\nu_1)I\right] \ . \tag{2.47}$$

To obtain the dispersion relation, we first consider the following quantity

$$\begin{aligned} P_t^2 - P_r^2 - P_y^2 &= \frac{1}{(\pi\alpha'\alpha)^2}(\kappa^2 - v_r^2 - v_y^2)\ I^2 \\ &= -\frac{1}{(\pi\alpha'\alpha)^2}\frac{\lambda_1\nu_1 c}{\beta}\ I^2 \end{aligned} \tag{2.48}$$

where the Virasoro constraints are used in the last equation. From the definitions of charges, $\nu_1 c\ I^2$ in the above equation can be determined in terms of other charges and the angle difference

$$-\frac{\lambda_1\nu_1 c}{\beta}\ I^2 = \frac{(\pi\alpha'\alpha)^2}{\lambda_1}\left[P_{\phi_1}^2 + \frac{\beta^4-\alpha^4}{2\alpha^2\beta^2}P_{\phi_1}^2 - \frac{2\beta}{\alpha}P_{\phi_1}(P_{\psi_1} - T_1\Delta\phi_1) + (P_{\psi_1} - T_1\Delta\phi_1)^2\right] . \tag{2.49}$$

Finally, we obtain the dispersion relation

$$\begin{aligned} P_t^2 - P_r^2 - P_y^2 &= \frac{1}{\lambda_1}\left[P_{\phi_1}^2 + \frac{\beta^4-\alpha^4}{2\alpha^2\beta^2}P_{\phi_1}^2 - \frac{2\beta}{\alpha}P_{\phi_1}(P_{\psi_1} - T_1\Delta\phi_1) + (P_{\psi_1} - T_1\Delta\phi_1)^2\right] \\ &= \frac{1}{\lambda_1}\left[\left(1 - \frac{\alpha^4+\beta^4}{2\alpha^2\beta^2}\right)P_{\phi_1}^2 + (\frac{\beta}{\alpha}P_{\phi_1} + T_1 p - P_{\psi_1})^2\right] , \end{aligned} \tag{2.50}$$

where $T_1 = \frac{\lambda_1}{2\pi\alpha'}$ and we have identified the angle difference $\Delta\phi_1$ with the world sheet momentum p.

2.2 Magnon solutions on $R \times S_1^3 \times S_2^3$

The equations of motion for ϕ_i and ψ_i $(i = 1, 2)$ are summarized as

$$\begin{aligned} h_i' &= \frac{1}{\lambda_i(\alpha^2-\beta^2)\sin^2\theta_i}(C_i - \alpha\lambda_i\omega_i\cos^2\theta_i + \beta\lambda_i\nu_i\sin^2\theta_i) , \\ g_i' &= \frac{1}{\lambda_i(\alpha^2-\beta^2)\cos^2\theta_i}(D_i + \alpha\lambda_i\nu_i\cos^2\theta_i + \beta\lambda_i\omega_i\cos^2\theta_i) \ . \end{aligned} \tag{2.51}$$

(2.32) becomes

$$\sum_{i=1}^{2}\lambda_i\theta_i'^2 = \frac{\alpha^2+\beta^2}{(\alpha^2-\beta^2)^2}(\kappa^2 - v_r^2 - v_y^2) - \sum_{i=1}^{2}K_i(\theta_i), \tag{2.52}$$

where

$$\begin{aligned} K_i(\theta_i) = \frac{1}{(\alpha^2-\beta^2)^2}\Bigg[\frac{\lambda_i\alpha^2}{\sin^2\theta_i}(\nu_i^2\sin^2\theta_i + \omega_i^2\cos^2\theta_i) + \Bigg[\frac{C_i^2}{\lambda_i\sin^2\theta_i} + \frac{D_i^2}{\lambda_i\cos^2\theta_i} \\ + \frac{2\alpha}{\sin^2\theta_i}(D_i\nu_i\sin^2\theta_i - C_i\omega_i\cos^2\theta_i)\Bigg]\Bigg] . \end{aligned} \tag{2.53}$$

Without loss of generality, we can set

$$\lambda_i\theta_i'^2 + K_i(\theta_i) \equiv \lambda_i\Gamma_i\,, \tag{2.54}$$

where Γ_i are some constants satisfying $\lambda_1\Gamma_1 + \lambda_2\Gamma_2 = \frac{\alpha^2+\beta^2}{(\alpha^2-\beta^2)^2}(\kappa^2 - v_r^2 - v_y^2)$.

When we set $D_i = 0$ and $C_i = \lambda_i c_i$, θ_i' are given by

$$\theta_i' = A_i \, \frac{\sqrt{(\sin^2\theta_i - \sin^2\theta_{i,min})}}{\sin\theta_i}, \tag{2.55}$$

where

$$\begin{aligned} A_i^2 &= \Gamma_i - \frac{\alpha^2}{(\alpha^2-\beta^2)^2}(\nu_i^2 - \omega_i^2) + \frac{2\alpha c_i \omega_i}{(\alpha^2-\beta^2)^2}\,, \\ \sin^2\theta_{i,min} &= \frac{(\alpha\omega_i - c_i)^2}{A_i^2}. \end{aligned} \tag{2.56}$$

In this background, the number of the magnon-like solution in each sphere is given by the similar relation in eq. (2.40)

$$\pi = \frac{2n_i}{\alpha}\int_{\theta_{i,min}}^{\pi/2} \frac{d\theta_i}{\theta_i'} = \frac{n_i \pi}{\alpha A_i}, \tag{2.57}$$

where n_i means the number of the magnon-like solution in i-th sphere. From the above equation, the number of the magnon becomes $n_i = \alpha A_i$. If we set the ratio between magnon-like numbers as $r \equiv \frac{n_2}{n_1}$, then this ratio is given by

$$r = \frac{A_2}{A_1}\ . \tag{2.58}$$

From now on, we set a magnon-like solution as one in the first sphere, which corresponds to r magnon-like solutions in the second sphere.

The conserved charges for a magnon-like solution are

$$\begin{aligned} P_t &= \frac{1}{\pi\alpha'}\frac{\kappa}{\alpha}\ I_1, &(2.59)\\ P_y &= \frac{1}{\pi\alpha'}\frac{v_y}{\alpha}\ I_1, &(2.60)\\ P_r &= \frac{1}{\pi\alpha'}\frac{v_r}{\alpha}\ I_1, &(2.61)\\ P_{\phi_i} &= -\frac{\lambda_i}{\pi\alpha'}\frac{1}{\alpha^2-\beta^2}\left(\frac{\beta}{\alpha}c_i + \alpha\nu_i\right)\ I_i, &(2.62)\\ P_{\psi_i} &= -\frac{\lambda_i}{\pi\alpha'}\frac{c_i - \alpha\omega_i}{\alpha^2-\beta^2}\left(I_i - I_i'\right)\,, &(2.63)\\ \Delta\phi_i &= \frac{2}{\alpha^2-\beta^2}\left[(c_i - \alpha\omega_i)I_i' + (\alpha\omega_i + \beta\nu_i)I_i\right], &(2.64) \end{aligned}$$

where

$$\begin{aligned} I_1 &= \int_{\theta_{1,min}}^{\pi/2} d\theta_1\, \frac{1}{\theta_1'} = \frac{\pi}{2A_1}, \\ I_1' &= \int_{\theta_{1,min}}^{\pi/2} d\theta_1\, \frac{1}{\theta_1' \sin^2\theta_1} = \frac{\pi}{2B_1}\,, \end{aligned}$$

$$I_2 = r\int_{\theta_{2,min}}^{\pi/2} d\theta_2\, \frac{1}{\theta_2'} = \frac{r\pi}{2A_2},$$
$$I_2' = r\int_{\theta_{2,min}}^{\pi/2} d\theta_2\, \frac{1}{\theta_2' \sin^2\theta_2} = \frac{r\pi}{2B_2}\,, \tag{2.65}$$

with $B_i^2 = (\alpha\omega_i - c_i)^2$. Then the relation (2.31) $\kappa^2 - v_r^2 - v_y^2 = \frac{1}{\beta}(-\nu_1 C_1 - \nu_2 C_2)$ can be rewritten in terms of charges

$$\begin{aligned} P_t^2 - P_r^2 - P_y^2 &= \sum_{i=1}^{2} \frac{1}{\lambda_i}\Bigg[P_{\phi_i}^2 + \frac{\beta^4-\alpha^4}{2\alpha^2\beta^2}P_{\phi_i}^2 - \frac{2\beta}{\alpha}P_{\phi_i}(P_{\psi_i} - T_i\Delta\phi_i) \\ &\qquad\qquad + (P_{\psi_i} - T_i\Delta\phi_i)^2\Bigg] \\ &= \sum_{i=1}^{2}\frac{1}{\lambda_i}\left[\left(1 - \frac{\alpha^4+\beta^4}{2\alpha^2\beta^2}\right)P_{\phi_i}^2 + \left(\frac{\beta}{\alpha}P_{\phi_i} + T_i\Delta\phi_i - P_{\psi_i}\right)^2\right], \end{aligned} \tag{2.66}$$

where $T_i = \frac{\lambda_i}{2\pi\alpha'}$ with $i = 1, 2$. This corresponds to the dispersion relation for the string soliton on the I-brane background when the it moves simultaneously on both the spheres, and this does not depend on the previous parameterization, Γ_i.

3. Discussion

In this paper we have studied the solutions for rotating strings in the background generated by a 1+1 dimensional intersection of two stacks of five branes in type IIB string theory. We have solved the motion of rotating string in this background and have analyzed the dispersion relation among various conserved charges. We have taken advantage of the fact there exists a parameter space where the motion in the two spheres effectively decoupled, and one could study the single magnon like solution in this background. Knowing the results of the present paper, it would be tempting to study the corresponding states in the dual theory exactly in case of the $\mathrm{AdS}_5\times\ \mathrm{S}^5$ background. It would certainly be interesting to check whether these magnon solutions are BPS from the bulk theory view point, which will give us clue about the boundary operators. We wish to come back to this issue in future.

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